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DISCUSSION PAPER

Vertical Ownership Without Control

by

Patrick Greenlee[*] and Alexander Raskovich[*]

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[*] U.S. Department of Justice, patrick.greenlee@usdoj.gov, and alexander.raskovich@usdoj.gov.
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Abstract

When a monopolist sells an input to an oligopoly, consumer and total surplus frequently are invariant to changes in passive ownership of the monopolist by downstream firms. Within broad classes of ownership profiles, strong invariance holds: the input and output choices of downstream firms are invariant to a change within the class. While passive ownership raises input demand, the upstream firm responds by raising price. Strong invariance always holds for bilateral monopoly. For a broad class of models with fixed proportions technologies, aggregate output is invariant across all passive ownership profiles. Passive ownership is privately profitable because it shifts sales from rivals.

1. Introduction

We explore passive partial ownership of an upstream firm by downstream firms. A prominent recent example is the passive 9% stake held by TCI, the largest U.S. cable television system operator, in Time Warner, a content producer. Such ownership arrangements also arise in automobile manufacturing, biotechnology, software and other industries. Abstracting from issues of potential foreclosure and incomplete information, we pose a basic question about ownership interests across vertically related firms. Since Spengler's (1950) seminal work, it has been well known that vertical integration can improve efficiency by eliminating double marginalization. Does partial ownership likewise mitigate double marginalization, at least in part? The answer depends on who owns whom and whether the ownership interest conveys control over pricing. When an upstream firm with market power acquires equity in a downstream firm with market power, it internalizes, at least partly, the adverse effect high input prices have on downstream profits. In contrast, when downstream firms own passive equity interests in an upstream firm, equilibrium quantities of inputs and outputs may remain unchanged. While an increase in a firm's passive interests in the upstream firm shifts its input demand curve outward, the upstream firm profitably responds by raising the input price. Perhaps surprisingly, these effects exactly cancel in a broad range of circumstances.

We study a game played in three stages. First downstream firms acquire passive ownership interests in an upstream monopolist. The monopolist then sets its input price. Finally, downstream firms choose their actions (*e.g.*, prices or quantities) and purchase the input combinations that produce their equilibrium outputs at minimum cost. We begin by taking the profile of passive ownership interests from stage one as given, and explore how changes in this profile affect the equilibrium of the continuation game. We then explore the scope for trade in passive backward interests in stage one, given that firms anticipate the effects changes in this profile will have on the equilibrium of the continuation game.

In both Cournot and Bertrand settings with rather weak restrictions on demand and production technologies, we show that "strong invariance" holds within broad classes of passive backward ownership profiles. That is, the input and output choices of every downstream firm are invariant across all ownership profiles within the class. One such invariance class is the set of uniform ownership profiles when downstream firms are otherwise symmetric. As a corollary, strong invariance always holds for bilateral monopoly. Our assumption that downstream firms acquire passive interests in the upstream firm prior to the upstream firm setting its input price is critical to these invariance results. This assumption is reasonable given that equity interests typically adjust less frequently than input prices. Less realistic, perhaps, is our assumption (following Greenhut and Ohta [1979], Salinger [1988], Flath [1989], and others) that the oligopolists are price-takers with respect to purchasing the monopolist's intermediate good.[1]

For a broad class of fixed proportions technologies that allows for linear or quadratic idiosyncratic costs, we show that aggregate equilibrium output is invariant across all ownership profiles in both a homogeneous Cournot and a symmetrically differentiated Bertrand setting. However, in these settings acquiring a passive backward interest is always profitable for a downstream firm that faces rivals, because the input price response by the upstream firm creates a cost advantage for the acquirer, allowing the acquirer to capture greater market share. We show that a privately held upstream firm also finds it advantageous to sell some passive interests to downstream firms. For a given level of aggregate interests, we show that the distribution of these interests across downstream firms resulting from efficient trade among them achieves allocative efficiency in the homogenous goods Cournot model, but is generally inefficient in the Bertrand model with symmetrically differentiated goods. The difference is that, with differentiated goods, consumer surplus depends on the distribution of production across firms, and downstream firms fail to fully internalize the consumer surplus effects of their trade in passive backward interests.

The remainder of the paper is organized as follows. We introduce the general model and derive the strong invariance result in Section 2. In Section 3, we specify a class of fixed proportions technologies and further examine Cournot and Bertrand competition. We investigate the incentives of downstream firms to acquire passive interests in the upstream firm, the willingness of a privately held upstream firm to sell passive interests in itself, and

[1] This distinguishes the monopolist's customers as "downstream" firms. See Salinger (1989) on the meaning of upstream and downstream.

characterize the equilibrium allocation of outstanding interests when downstream firms can costlessly trade stakes among themselves. Section 4 concludes.

2. Strong Invariance

Each of n downstream firms, indexed by i, produces output q_i according to a production function $q_i = f_i(x_i, y_i)$ that exhibits non-increasing returns to scale. The price of input y is exogenous and normalized to one. An upstream monopolist produces input x at constant marginal cost $c \geq 0$ and sets a uniform price m. Let ω_i denote firm i's ownership interest in the upstream firm and let $\omega = (\omega_1,...,\omega_n)$ be the profile of interests held by all downstream firms. We assume ownership stakes held by downstream firms are claims on upstream profits that convey no control over upstream pricing. Bresnahan and Salop (1986) label such claims "silent financial interests" in the context of partial cross-ownership among rival firms. While the assumption of no control is plausible only when ownership interests are small, the only formal restrictions on ownership interests we impose are $\omega_i \geq 0$ for all i and $\sum \omega_i \leq 1$. Why backward ownership interests may remain small and passive is beyond the scope of the paper.[2]

Let θ_i be downstream firm i's action, and let $\theta = (\theta_1,...,\theta_n)$ be the profile of downstream actions. As part of a general formulation, θ may represent a profile of prices, output levels, or some other action such as advertising intensity or investment level. Each downstream firm i earns profit of

$$\pi_i(\theta,m,\omega) \;=\; p_i(\theta)\,q_i(\theta) + \omega_i\,\Pi(\theta,m,\omega) - m\;x_i^*(\theta,m,\omega) - y_i^*(\theta,m,\omega) \qquad (1)$$

where $p_i(\theta)$ and $q_i(\theta)$ are firm i's price and quantity, $x_i^*(\theta,m,\omega)$ and $y_i^*(\theta,m,\omega)$ are input levels that produce $q_i(\theta)$ at minimum cost, and $\Pi(\theta,m,\omega) = (m-c)\sum x_j^*(\theta,m,\omega)$ is upstream profit. The input demand functions $x_i^*(\theta,m,\omega)$ and $y_i^*(\theta,m,\omega)$ take into account the perceived discounts attributable to the downstream firm's partial ownership of the upstream firm. With respect to the output demand functions $q_i(\theta)$, we assume that the final

[2] Riordan (1991) and Dasgupta and Tao (2000) examine incomplete information models where agency problems limit the degree of passive ownership in an upstream firm.

goods are substitutes, but allow them to be homogeneous or differentiated. The first order conditions for profit maximization are

$$\frac{\partial \pi_i}{\partial \theta_i} = q_i \frac{\partial p_i}{\partial \theta_i} + p_i \frac{\partial q_i}{\partial \theta_i} - \left(m \frac{\partial x_i^*}{\partial q_i} + \frac{\partial y_i^*}{\partial q_i} \right) \frac{\partial q_i}{\partial \theta_i} + \omega_i \left(m - c \right) \sum_j \frac{\partial x_j^*}{\partial q_j} \frac{\partial q_j}{\partial \theta_i} = 0 , \tag{2}$$

where $\partial q_j / \partial \theta_i$ represents the change in firm *j*'s unit sales when θ_i increases, holding fixed the actions of firm *i*'s rivals.

Equation (2) can be written as

$$\frac{\partial \pi_i}{\partial \theta_i} = q_i \frac{\partial p_i}{\partial \theta_i} + p_i \frac{\partial q_i}{\partial \theta_i} - \left(s_i \frac{\partial x_i^*}{\partial q_i} + \frac{\partial y_i^*}{\partial q_i} \right) \frac{\partial q_i}{\partial \theta_i} = 0, \tag{3}$$

where

$$s_i = m - \omega_i (m - c) \Gamma_i , \tag{4}$$

$$\Gamma_i = \left(\sum_j \frac{\partial x_j^*}{\partial q_j} \frac{\partial q_j}{\partial \theta_i} \right) \Bigg/ \left(\frac{\partial x_i^*}{\partial q_i} \frac{\partial q_i}{\partial \theta_i} \right). \tag{5}$$

Γ_i is the sum of own and cross effects on aggregate input consumption $X = \sum x_j^*$ flowing from firm *i*'s employment of input *x* at the margin, holding rival actions fixed. In equations (3) and (4), s_i can be interpreted as firm *i*'s effective cost of input *x*, inclusive of the opportunity costs of employing *x* optimally. For Cournot competition ($\theta_i = q_i$), equation (5) simplifies to $\Gamma_i = 1$, since by assumption $\partial q_j / \partial q_i = 0$ for $j \neq i$. Firm *i*'s effective cost of input *x* is then reduced below the explicit price *m* by the rebate $\omega_i (m - c)$ implied by firm *i*'s ownership stake in the upstream firm. For Bertrand competition ($\theta_i = p_i$), firm *i* obtains the rebate $\omega_i (m - c)$ on its own purchases of *x*, but in addition $\partial q_j / \partial p_i > 0$ for $j \neq i$ in equation (5). These cross effects represent an opportunity cost to firm *i* of employing *x*. Expanding its employment of *x* requires firm *i* to lower its price, so that the additional output will be sold. This decreases demand for rival final goods and hence lowers rival consumption of *x*, thereby tending to reduce firm *i*'s take in upstream profits.

We assume that for any input price *m* and ownership profile ω, the downstream game has a unique interior Nash equilibrium $\theta(m, \omega)$. Let $x_i(m, \omega) = x_i^*(\theta(m, \omega), m, \omega)$ be firm *i*'s equilibrium consumption of *x* and let $\Gamma_i(m, \omega)$ be given by equation (5) evaluated at

$\theta(m,\omega)$. Given the downstream equilibrium, upstream profits are then given by $\Pi(m,\omega)=(m-c)\sum x_i(m,\omega)$, and the first order condition for an upstream maximum is

$$\frac{\partial \Pi(m,\omega)}{\partial m}=\sum_i\left(x_i(m,\omega)+(m-c)\frac{\partial x_i(m,\omega)}{\partial m}\right)=0\,. \tag{6}$$

Firm i's effective unit cost of input x in equilibrium is

$$s_i(m,\omega)=m-\omega_i(m-c)\Gamma_i(m,\omega)\,. \tag{7}$$

Note that $\partial x_i(m,\omega)/\partial m$ can be written as $\sum_j\left(\partial x_i(m,\omega)/\partial s_j\right)\left(\partial s_j(m,\omega)/\partial m\right)$ in equation (6). If $\partial\Gamma_j(m,\omega)/\partial m=0$ for all j, then by equation (7) $\partial s_j(m,\omega)/\partial m=1-\omega_j\,\Gamma_j(m,\omega)$. In this case, the upstream first order condition (6) can be written as

$$\frac{\partial \Pi(m,\omega)}{\partial m}=\sum_i\left(x_i(m,\omega)+\sum_j\left(s_j(m,\omega)-c\right)\frac{\partial x_i(m,\omega)}{\partial s_j}\right)=0\,. \tag{8}$$

Now consider a change in the profile of ownership interests from ω' to ω''. Given ω', let m' be the solution to equation (8) maximizing upstream profit, and let s', q', x' and y' be the corresponding equilibrium vectors of effective input costs, outputs, and employed inputs. Define m'', s'', q'', x'' and y'' similarly for ω''. Downstream input decisions depend on m and ω only insofar as these variables change s, so that when $s''=s'$, the downstream and upstream first order conditions (3) and (8) continue to hold at the initial equilibrium values: $x''=x'$, $y''=y'$ and $q''=q'$. We call this strong invariance. To summarize, conditions (9) and (10) are jointly sufficient for strong invariance:

$$\left.\frac{\partial\Gamma_i(m,\omega')}{\partial m}\right|_{m=m'}=0,\ \text{for } i=1,\ldots,n, \tag{9}$$

there exists an m'' satisfying $s_i(m'',\omega'')=s_i(m',\omega')$ for $i=1,\ldots,n.$ (10)

Condition (9) holds fairly broadly. It always holds for Cournot competition ($\Gamma_i=1$), and holds for Bertrand competition when production technologies require that the monopolized input be used in fixed proportion with output (in which case $\partial^2 x_i^*/\partial q_i\partial m=0$).[3] For these

[3] For the Bertrand setting, condition (9) does not require firms to have symmetric technologies. Firms may have varying idiosyncratic costs, as in the models of Section 3, and may also differ in the fixed proportion of the monopolized input to final output.

two sets of cases, the equilibrium values $\Gamma_i(m,\omega)$ are everywhere invariant to *m*. Condition (9) is weaker than this, requiring only that the derivative of $\Gamma_i(m,\omega')$ with respect to *m* be zero at the upstream optimum m'.

By equation (7), condition (10) implies that m'' must satisfy

$$m'' = \left(\frac{1-\omega_i'\Gamma_i}{1-\omega_i''\Gamma_i}\right)(m'-c) + c, \text{ for } i=1,\dots,n, \tag{11}$$

where $\Gamma_i = \Gamma_i(m',\omega') = \Gamma_i(m'',\omega'')$. Equations (11) hold if and only if there exists an $r > 0$ satisfying $1-\omega_i'\Gamma_i = r\,(1-\omega_i''\Gamma_i)$ for all *i*. Given a profile ω', letting *r* vary and solving this system of equations for ω'' traces out the invariance class to which ω' belongs. One such invariance class is the set of uniform ownership profiles, when firms are otherwise symmetric (*i.e.*, $\Gamma_i = \Gamma_j$ for all *i*, *j*).[4]

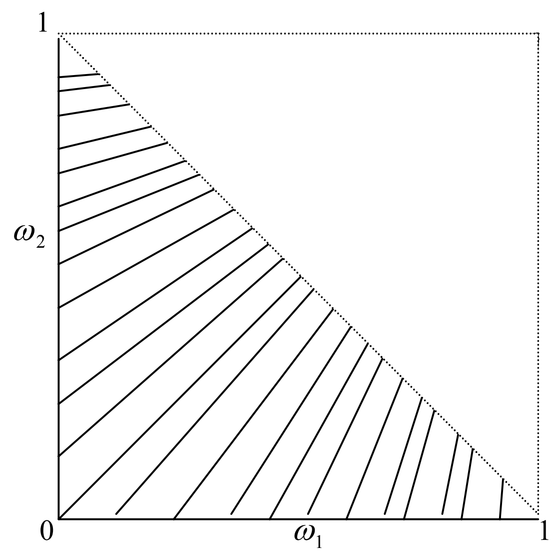

Figure 1: *Some strong invariance rays for Cournot duopoly.*

For the case of downstream Cournot duopoly, Figure 1 illustrates some of the rays in (ω_1,ω_2) space along which strong invariance holds.

For the case of bilateral monopoly ($n=1$), $\Gamma = 1$ and equation (11) can always be satisfied, so strong invariance holds for all passive ownership interests held by the

[4] Flath (1989) shows that the overall price-cost margin in a successive Cournot oligopoly is unaffected by a symmetric change to uniform, passive backward ownership interests when

downstream firm. In this case, net upstream profit (after paying the downstream shareholder) does not change with the ownership interest:

$$(1-\omega'')\Pi'' \;=\; (1-\omega'')(m''-c)X \;=\; (1-\omega')(m'-c)X \;=\; (1-\omega')\Pi'. \tag{12}$$

The upstream firm is thus willing to sell shares to the downstream firm at any non-negative premium over the equilibrium value of the upstream profits claimed by the shares. The downstream firm, however, has no incentive to acquire a passive interest at any such premium. This is for two reasons. First, an ownership stake would cause the upstream firm to raise the input price m to the level where the effective input price s remains unchanged. Second, the downstream firm would receive no upstream profit beyond its own rebate. A solitary downstream firm bears the full brunt of the price increase induced by the acquisition of a passive interest. In the following section, we show that acquiring a passive interest is profitable for a downstream firm that faces rivals because the upstream firm's price response creates an effective cost advantage for the acquiring firm.

3. Fixed Proportions Technology

In this section, we examine in turn Cournot and Bertrand settings in which each firm requires one unit of the monopolized input to produce one unit of output. Throughout this section we assume there are $n \geq 2$ downstream firms with cost functions of the form

$$C_i(q_i) = (m+k_i)\, q_i + \tfrac{a}{2}\, q_i^2\,, \tag{13}$$

where $a, k_i \geq 0$. Firm i's marginal cost is linear, with intercept $m+k_i$ and slope a. Sections 3.1 to 3.3 examine homogeneous good Cournot competition, while Sections 3.4 and 3.5 consider a linear Bertrand environment in with symmetrically differentiated goods. Throughout this Section, we assume that ownership changes induce no entry or exit of downstream firms, reflecting significant barriers to entry.

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demand is isoelastic and downstream firms employ identical constant returns to scale technology. This is a special case of our strong invariance result.

3.1 *Cournot Competition*

Assume that firms produce homogeneous goods and choose quantities ($\theta_i = q_i$). The profit of firm *i* can be expressed as

$$\pi_i(q_i) \;=\; P(Q)\, q_i - \tfrac{a}{2} q_i^{\,2} - q_i\left(k_i + m(1-\omega_i) + \omega_i\, c\right) + \omega_i\, \Pi_{-i}\,, \tag{14}$$

where Q is aggregate output, $P(Q)$ the downstream market clearing price, and $\Pi_{-i} = (m-c)\sum_{j\neq i} x_j$. The downstream first order conditions are

$$P(Q) + P'(Q)\, q_i - a\, q_i - k_i - m(1-\omega_i) - \omega_i\, c \;=\; 0. \tag{15}$$

Letting $\Omega = \sum \omega_i$, $K = \sum k_i$, and summing equations (15) across downstream firms yields

$$n\, P(Q^e) + P'(Q^e)\, Q^e - a\, Q^e \;=\; m(n-\Omega) + K + \Omega c\,, \tag{16}$$

which implicitly defines the equilibrium aggregate output Q^e. In the downstream subgame, the right hand side of equation (16) is constant, so imposing the weak stability condition that the left hand side is decreasing in Q^e implies a one-to-one mapping between *m* and Q^e, for fixed Ω.[5] Let $m(Q^e,\Omega)$ denote this function. Since each unit of output requires one unit of input *x*, $m(\cdot,\Omega)$ is also the inverse demand for *x*, given Ω.

It is convenient to characterize the upstream firm's problem as selecting Q^e to maximize

$$\Pi \;=\; \left(m(Q^e,\Omega) - c\right) Q^e \;=\; \left(\frac{n\, P(Q^e) + Q^e\, P'(Q^e) - aQ^e - K - n\, c}{n-\Omega}\right) Q^e\,. \tag{17}$$

Observe that passive ownership interests appear only as a scaling factor in equation (17). Therefore the upstream firm's optimal choice of Q^e and consumer surplus are invariant to passive ownership interests. Keeping this invariance in mind, note by inspection of equation (17) that the optimal input price $m(Q^e,\Omega)$ is an increasing function of aggregate interests Ω, but does not depend on the allocation of these interests across downstream firms.

[5] This is similar to the condition in Seade (1980) and Bergstrom and Varian (1985).

3.2 Cournot: Acquiring Passive Interests

We now turn to the effects of an increase in passive ownership by a single downstream firm, which for the moment we label firm 1: $\omega'' = \omega' + (\varepsilon, 0, ..., 0)$, $\varepsilon > 0$. Recall that $\Gamma_i = 1$ for Cournot competition, so by equation (7) the effective input cost of firm *i* is

$$s_i(m, \omega_i) = m - \omega_i(m - c). \tag{18}$$

Since firm 1's additional passive ownership raises the upstream firm's optimal input price from m' to m'', the effective input cost of each of firm 1's rivals rises. In contrast, we establish in the appendix that firm 1's effective input cost declines. That is, $s_1(m'', \omega_1'') < s_1(m', \omega_1')$ and $s_j(m'', \omega_j'') > s_j(m', \omega_j')$ for $j > 1$.

Rearranging terms in equations (15), equilibrium outputs can be expressed as

$$q_i(m, \omega) = \frac{P(Q^e) - s_i(m, \omega_i) - k_i}{a - P'(Q^e)}. \tag{19}$$

Thus the reduction in firm 1's effective input cost causes its equilibrium quantity to increase at the expense of each of its rivals. By equation (18), rivals of firm 1 with identical passive interests face the same effective input costs and thus lose the same market share as a result of firm 1 increasing its passive ownership stake.[6]

Industry profits, and hence total surplus, may rise or fall depending on the relative efficiency of firm 1, as determined by k_1 and *a*. When firms have identical constant returns to scale technology ($a = 0$, $k_i = k$ for all *i*), aggregate production costs are invariant to shifting output among firms, hence industry profits and total surplus are invariant across all passive ownership profiles. When $a = 0$ and all (possibly heterogeneous) firms $i \neq 1$ have identical holdings in the upstream firm, industry profits and total surplus rise when firm 1 increases its ownership stake if and only if firm 1 is more efficient than average. For this case aggregate production costs are $\sum (c + k_i) q_i$. If δ is the increase in firm 1's equilibrium quantity after acquiring the passive interest ($q_1(m'', \omega'') - q_1(m', \omega') = \delta > 0$), then

[6] Riordan (1998) finds a similar outcome--increased market share for the acquirer and higher input prices to rivals--when a dominant firm vertically integrates (*with* control) into a competitive upstream industry. In our setting, *m* rises due to opportunistic behavior by an upstream monopolist, not by a downstream firm seeking to foreclose rivals.

$q_j(m'',\omega'') - q_j(m',\omega') = -\frac{\delta}{n-1}$ for each firm j > 1, and the change in aggregate production cost is $\frac{n\delta}{n-1}(k_1 - \bar{k})$, where $\bar{k} = \frac{1}{n}\sum k_i$. In this case total surplus increases if and only if $k_1 < \bar{k}$.

In contrast to the bilateral monopoly case discussed in Section 2, a downstream firm always gains from additional passive interests in the upstream firm in the present setting. We establish in the appendix that

$$\frac{\partial \pi_i}{\partial \omega_i} > 0. \tag{20}$$

The burden of an input price increase induced by the acquisition of a passive interest in the upstream firm is borne by all downstream firms, not just the acquirer. Because the acquirer is the only firm to receive the additional rebate, the acquirer reduces its costs while raising rivals' costs.

3.3 Cournot: Trading Passive Interests

Having established that each downstream firm benefits from the acquisition of a passive interest in the upstream firm, we now consider whether an upstream firm that is initially held privately would find it advantageous to sell passive interests to downstream firms. Gains to such trade exist if the profit gain to the downstream firm exceeds the upstream firm's lost profit from selling the passive interest. Formally, we expect trade if

$$\frac{\partial\big((1-\Omega)\,\Pi\big)}{\partial \omega_i} + \frac{\partial \pi_i}{\partial \omega_i} > 0. \tag{21}$$[7]

While the upstream firm's margin increases with an additional passive stake, it retains a smaller fraction of each sale and the first term in condition (21) is negative. From above, the second term in (21) is always positive. In the appendix we establish that condition (21) is satisfied at $\Omega = 0$. That is, the upstream firm always finds it profitable to sell some passive interests to downstream firms.

[7] If initially the upstream firm is publicly held with some shares owned by entities unrelated to the upstream or downstream markets, we expect such owners to trade with downstream firms, since by condition (20) these firms would be willing to pay independent investors a premium for their shares.

Let $\Omega > 0$ be the aggregate passive interests in the upstream firm. Explaining the magnitude of Ω, as well as why the ownership interests remain passive, is beyond the scope of the present analysis. We turn now to the distribution of Ω across downstream firms. Recall that upstream profits are invariant to this distribution, so it is reasonable to assume the upstream firm would allow stakes to be tradable. We examine the distribution of outstanding interests that results from trade among downstream firms when interests are infinitely divisible and can be costlessly traded, and when trade proceeds until all gains from trade have been exploited.

Aggregate equilibrium output Q^e is invariant to aggregate passive interests Ω, and the upstream firm's optimal input price $m(Q^e, \Omega)$ is invariant to the distribution across downstream firms of interests summing to a given Ω. For fixed Ω, the equilibrium output of firm i can be written as a function of ω_i, $q_i(\omega_i) = q_i(m(Q^e, \Omega), \omega)$, as determined by equations (18) and (19). For a given interest ω_i, firm i's equilibrium social marginal cost of production is

$$t_i(\omega_i) = c + k_i + a\, q_i(\omega_i). \tag{22}$$

We establish in the appendix that firms i and j can increase their joint surplus by transferring some passive interests from j to i if and only if $\omega_j > 0$ and $t_i(\omega_i) < t_j(\omega_j)$. Therefore an ownership profile ω^* for which all such gains from trade have been exhausted satisfies

$$t_i(\omega_i^*) = t_j(\omega_j^*), \qquad \text{for all } i, j, \text{ such that } \omega_i^*, \omega_j^* > 0, \tag{23}$$

and $t_i(\omega_i^*) \le t_j(0)$ for any pair of firms such that $0 = \omega_j^* < \omega_i^*$. For a given Ω, these conditions imply that the ownership profile ω^* maximizes allocative efficiency. Total surplus is also maximized by ω^* given Ω. Recall that consumer surplus and upstream profits are both invariant to the distribution of interests ω, holding aggregate interests Ω fixed. Moreover, trade in interests between downstream firms i and j has no spillover effects on the profits of downstream firms $k \ne i, j$, since such trade leaves $q_k(\omega_k)$, $P(Q^e)$, and $m(Q^e, \Omega)$ unchanged. Therefore the change in joint profits of firms i and j from a transfer of passive interests between them equals the change in total surplus from such a trade.

Since the most efficient downstream firms have the largest market shares when $\Omega = 0$ and they hold larger passive interests than their less efficient rivals when $\Omega > 0$, an increase in Ω raises concentration in the downstream market while reducing production costs. Consumer surplus does not change, but producer surplus increases, so passive backward investment is an economic environment in which total surplus and measures of concentration such as the Herfindahl-Hirschman Index move in the same direction.[8]

When downstream firms employ identical decreasing returns to scale technology ($a > 0$, $k_i = k$ for all *i*), they face a prisoner's dilemma if confronted with a prospective sale of passive interests in the upstream firm. Assume that the upstream firm sells an initial block Ω of passive interests to the downstream oligopoly. If all gains from trade have been exhausted in the ultimate distribution ω of outstanding interests, then ω satisfies $t_i(\omega_i) = t$, and hence $\omega_i = \Omega / n$, for all *i*.[9] Recall from Section 2 that strong invariance holds across uniform ownership profiles when firms are otherwise symmetric. Thus the Cournot equilibrium for any $\Omega > 0$ is identical to the Cournot equilibrium for $\Omega = 0$. Yet by condition (20), the seller can generate positive revenue from the sale of passive backward interests if downstream firms act noncooperatively in their decisions to purchase these interests. Downstream firms collectively would be better off if they could engage in a concerted boycott of the sale of passive backward interests.

3.4 Bertrand Competition

We now assume that firms choose prices ($\theta_i = p_i$) and face the linear demand system

$$q_i = d_i - p_i + \gamma \sum_{j \neq i} p_j , \qquad \text{for } i = 1,...,n , \tag{24}$$

[8] Farrell and Shapiro (1990a, 1990b) emphasize this possibility for capital investment and mergers in oligopoly settings.

[9] Using (19) and (22) for this case, $t_i(\omega_i) < t_j(\omega_j)$ if and only if $\omega_i < \omega_j$. Thus, in equilibrium, all firms hold the same size stakes. When $a = 0$, t_i is constant across all firms and all ownership profiles, so there are never any gains to trade and all allocations of passive stakes are trading equilibria.

where $d_i, \gamma > 0$. Final goods are symmetrically differentiated in the sense that the own- and cross-price effects on demand in equations (24) are the same for all downstream firms. Demand intercepts d_i may vary across firms. Summing equations (24) yields

$$Q = D - \Gamma P, \tag{25}$$

where $Q = \sum q_i$, $D = \sum d_i$, $\Gamma = 1-(n-1)\gamma$ and $P = \sum p_i$. To assure that $\partial Q / \partial p_i = -\Gamma < 0$, we assume $\gamma < 1/(n-1)$.

Note that $\gamma > 0$ implies $\Gamma < 1$. Each downstream firm perceives that employing an additional unit of input, and lowering price sufficiently to sell the additional unit of output, depresses input usage of rival firms. Since $\Gamma = 1$ for Cournot competition, equation (4) implies that the effective input cost s_i is higher for Bertrand competitors than for similarly situated Cournot competitors. Recognition that lowering price has adverse effects on the demand for rival goods tends to dampen the incentive of a Bertrand competitor to cut price subsequent to acquiring a passive interest in the upstream firm. In this sense, passive backward ownership in Bertrand environments bears some similarity to passive cross-ownership among horizontal competitors.[10] In those horizontal settings, firms partially internalize the adverse effects that low pricing has on rival margins and quantities, and respond by raising price and increasing (downstream) margins. However, our vertical setting differs in two important respects. First, downstream firms treat the upstream margin as fixed. Passive backward interests thus give downstream firms an incentive to expand aggregate input usage by expanding aggregate output.[11] Second, the upstream firm optimally responds to an increase in passive backward ownership by raising the input price *m*. Thus despite the difference in Γ between Cournot and Bertrand settings, strong invariance holds across

[10] The literature examining passive ownership of horizontal rivals includes Bresnahan and Salop (1986), Gilo (2000), O'Brien and Salop (2000), Reitman (1994), and Reynolds and Snapp (1986).

[11] Our analysis likewise differs from Joskow and Tirole's (2000) study of transmission rights on electric power networks. Since transmission capacity that delivers power from another region is fixed and fully utilized in their model, ownership of financial transmission rights motivates a local generator in a power importing area to increase the margin earned on transmission (equal to the difference in regional prices for power). This is accomplished by raising the local price for power.

uniform ownership profiles in each when firms are otherwise symmetric, as we established in Section 2.

Returning to the present Bertrand setting, downstream firm i's profit is given by $p_i q_i + \omega_i (m-c)Q - C_i(q_i)$, where $(m-c)Q$ is upstream profit and total downstream production costs $C_i(q_i)$ are given by (13). The first order conditions for downstream firms can be written as

$$(1+a)q_i - p_i + k_i + s_i = 0, \tag{26}$$

where $s_i = m - \omega_i (m-c)\Gamma$ is the effective input cost facing firm i. With some work, the upstream first order condition can be written as

$$D - \left(2S + K - cn\right)\Gamma = 0, \tag{27}$$

where $K = \sum k_i$ and

$$S = \sum s_i = nm - \Omega(m-c)\Gamma. \tag{28}$$

Given that all other terms on the left hand side of equation (27) are constant, it follows that the equilibrium value of S is also constant across all ownership profiles. Moreover, by equation (28), the upstream firm's optimal input price m is an increasing function of aggregate passive interests Ω.

3.5 Bertrand: Acquiring and Trading Passive Interests

This Bertrand environment generates incentives to buy and sell passive stakes similar to the Cournot case above. When firm i acquires an additional passive interest from the upstream firm, the resulting increase in m raises rival effective input costs ($\partial s_j / \partial \omega_i > 0$, $j \neq i$) and, by the invariance of S, firm i's own effective input cost falls $(\partial s_i / \partial \omega_i < 0)$. This suggests that a downstream firm always finds it profitable to increase its passive stake in the upstream firm (condition (20)). In the appendix, we establish this result and also confirm that condition (21) holds at $\Omega = 0$ for this Bertrand environment: a privately held upstream firm finds it advantageous to sell some passive interests to downstream firms.

Summing together equations (26) and then applying equation (25), equilibrium aggregate output can be expressed as

$$Q^e = \frac{D-(K+S)\Gamma}{1+(1+a)\Gamma}. \tag{29}$$

Given that the upstream firm's optimal setting of *m* implies *S* is constant, equation (29) shows that, as in the Cournot case, aggregate output is invariant across all ownership profiles ω. Similarly, the upstream firm's optimal *m*, and hence equilibrium upstream profits, are also invariant to the distribution of passive interests across downstream firms. Proceeding as before, we take $\Omega > 0$ as exogenous and examine the distribution of interests that arises from costless trade among downstream firms.

In contrast to the Cournot model above, trade among downstream firms does not necessarily result in an efficient allocation of outstanding ownership shares in the present Bertrand setting. The difference is that final goods are assumed to be homogenous in the Cournot model of Sections 3.1-3.3, whereas they are differentiated here. Although in both settings aggregate output Q^e is invariant to the distribution of passive interests, consumer surplus also depends on the distribution of Q^e across downstream firms when final goods are differentiated. In their trading of passive interests, downstream firms fail to fully internalize effects on consumer surplus.

In the appendix, we establish the following two conditions for the Bertrand environment. Let the joint passive interests in the upstream firm held by downstream firms *i* and *j* be $\omega_i + \omega_j + \varepsilon$. The $\varepsilon > 0$ stake may be held by either firm as a result of trade. The joint profits of the firms would be higher when firm *i* holds ε if and only if

$$q_i^W - q_j^W > \frac{1+\gamma}{\gamma}(\omega_i - \omega_j)(m-c)\Gamma, \tag{30}$$

where q_k^W is firm *k*'s equilibrium quantity were it to hold interests totaling $\omega_k + \varepsilon$. Total surplus would be higher when firm *i* holds ε instead of firm *j* if and only if

$$q_i^W - q_j^W > \frac{1+\gamma}{1+2\gamma}(\omega_i - \omega_j)(m-c)\Gamma. \tag{31}$$

If $q_i^W > q_j^W$, then condition (31) holds whenever condition (30) does. In this case trade, when it occurs, improves total surplus. However, there may be total surplus gains that go unrealized because there are no mutually profitable trades of such stakes. If $q_i^W < q_j^W$, then condition (30) holds whenever condition (31) does. Trade may reduce total surplus in such cases.

When all gains from trade have been exhausted, inequality (30) cannot hold for any pair of downstream firms that own positive stakes in the upstream firm. For the fully symmetric case ($k_i = k$, $d_i = d$ for all i), (30) reduces to $\omega_i < \omega_j$. Since mutually beneficial trades exist whenever stake sizes differ in such settings, the equilibrium allocation is uniform ($\omega_i^* = \Omega/n$ for all i).[12] The strong invariance result of Section 2 then applies: the input and output choices of every downstream firm are exactly what they would have been for $\Omega = 0$. Yet, as in the Cournot case, downstream firms face a prisoner's dilemma in that the upstream firm can generate revenue by auctioning some passive interests to downstream firms bidding noncooperatively.

4. Conclusions

Economists, antitrust enforcers and regulators have long known that full vertical integration can improve efficiency by eliminating double marginalization. Our analysis reveals a more complex picture when vertical ownership interests are partial and passive.

For the classic case of successive monopoly with fixed or variable proportions, passive partial ownership of the upstream firm by the downstream firm has no effect on equilibrium inputs and outputs. Any such acquisition by the downstream monopolist is met by an exactly offsetting increase in input price by the upstream monopolist. The downstream monopolist thus cannot gain by acquiring a passive interest in the upstream monopolist at any positive share price. In contrast, acquiring a passive interest in the upstream monopolist always increases the downstream profits of the acquiring firm for the oligopoly environments examined in Section 3. This is because the resulting input price response by the upstream firm raises the costs of the acquirer's rivals, while lowering the acquirer's effective cost (net of the rebate implied by the ownership interest). Although aggregate equilibrium output remains constant in both the homogeneous Cournot and symmetrically differentiated

[12] For the fully symmetric case, inequality (31) also reduces to $\omega_i < \omega_j$ so trade always results in the allocation of outstanding interests that maximizes total surplus.

Bertrand examples, downstream firms can seek larger market share by acquiring passive interests in the upstream firm. When a given level of outstanding interests can be costlessly traded among downstream firms, the resulting distribution of interests after gains from trade have been exhausted achieves allocative efficiency in the homogenous Cournot model. In the symmetrically differentiated Bertrand model, however, the post-trade distribution of outstanding interests is generally inefficient, because downstream firms fail to fully internalize the consumer surplus effects of their trades. In Section 2, we show that there exist classes of passive ownership profiles such that the equilibrium input and output choices of every downstream firm are invariant within the class. In particular, strong invariance holds across all uniform profiles of passive ownership (including the zero vector), when downstream firms are otherwise symmetric. For our fixed proportions examples, we show that costless trade among *ex ante* identical firms results in the uniform distribution of outstanding shares. In such cases, by strong invariance the downstream equilibrium is exactly what it would have been had no downstream firm acquired any passive interests in the upstream firm. Yet the upstream firm can profit by selling passive backward interests to downstream firms fighting for market share.

Our analysis suggests that regulators and antitrust enforcers should not assume that passive backward ownership interests have effects on double marginalization similar to those of full vertical integration. Such interests may engender offsetting input price responses by the upstream firm. Viewed from another perspective, our analysis points to the importance of control to realizing double marginalization efficiencies from backward ownership. Out of concern that these rights could be wielded to foreclose rivals, regulators and antitrust enforcers have sometimes imposed restrictions on the control rights of owners of backward interests. For example, the Federal Communications Commission's channel occupancy rules generally prohibit a cable system operator from devoting more than 40% of its activated channel capacity to national video programming services in which the operator has an "attributable" interest. A cable operator has an attributable interest if the operator has a voting interest of at least 5% in the cable programmer. Non-voting ownership interests are not attributable, however, nor are the interests of "insulated" limited partners who must certify that they are not materially involved in the programming-related decisions of the partnership.[13] As another example, foreclosure concerns led the Federal Trade Commission to require that the 1995 deal involving Time Warner, Turner and TCI be restructured, so that the resulting ownership interest by TCI (a large cable operator) in Time Warner (a large cable programmer) would remain passive (see Pitofsky, 1997). Our analysis suggests that, in crafting safeguards against foreclosure concerns, regulators and antitrust enforcers should be aware of the possibility that stripping control rights from backward ownership interests might have adverse efficiency consequences.[14]

[13] See "Review of the Commission's Cable Attribution Rules," 14 FCC Rcd. 19,014 (released October 20, 1999).

[14] Contrary to our modeling assumptions of uniform linear pricing by the upstream firm, program carriage agreements between cable programmers and cable system operators typically are negotiated and involve complex price and non-price terms. For analyses of cable programming sales that assume efficient bargaining, see Chipty and Snyder (1999) and Raskovich (2000). We suspect that the actual bargaining is not efficient, and that moral hazard problems play a significant role in many vertical relationships. For these, backward vertical ownership interests may improve matters. See, for example, Riordan (1991) and Dasgupta and Tao (2000).

Appendix

<u>*The Cournot Fixed Proportions Model*</u>

Proof that $s_1(\omega'') < s_1(\omega')$ *whenever* $n \geq 2$.

Let $T = nP(Q^e) + P'(Q^e)Q^e - aQ^e - K$ for ease of notation. From (17), the equilibrium input price may be expressed as $m(\Omega) = (T - \Omega c)/(n - \Omega)$. Let $\Omega = \Sigma\,\omega_i'$. By (5), $s_1(\omega'') < s_1(\omega')$ if and only if

$$\frac{(1-\omega_1)(T-\Omega c)}{n-\Omega} - \frac{(1-\omega_1-\varepsilon)(T-\Omega c-\varepsilon c)}{n-\Omega-\varepsilon} - \varepsilon c > 0$$

The left hand side can be rewritten as

$$\frac{\varepsilon(n-1-\Omega+\omega_1)(T-cn)}{(n-\Omega)(n-\Omega-\varepsilon)} = \frac{(n-1-\Omega+\omega_1)}{(n-\Omega-\varepsilon)}\big(m(\Omega)-c\big)\,\varepsilon > 0 \qquad QED.$$

Proof that $\partial\pi_i/\partial\omega_i > 0$ *whenever* $n \geq 2$.

Using (17), we make the following calculations:

$$m-c = \frac{T-\Omega c}{n-\Omega} - c = \frac{T-cn}{n-\Omega} \qquad \frac{\partial m}{\partial\omega_i} = \frac{-c(n-\Omega)+T-\Omega c}{(n-\Omega)^2} = \frac{m-c}{n-\Omega} > 0$$

From (19), $\dfrac{\partial q_i}{\partial\omega_i} = \dfrac{-\partial s_i/\partial\omega_i}{a-P'(Q^e)} = \dfrac{-\frac{\partial}{\partial\omega_i}\big(m-\omega_i(m-c)\big)}{a-P'(Q^e)} = \dfrac{m-c}{a-P'(Q^e)}\left[1-\dfrac{1-\omega_i}{n-\Omega}\right] > 0$

Downstream profits can be expressed in the following manner (the rebate on own purchases is included in the second term)

$$\pi_i = q_i\big(P-m-k_i-\tfrac{1}{2}a\,q_i\big)+\omega_i\ (m-c)\,Q^e$$

$$\frac{\partial\pi_i}{\partial\omega_i} = \big(P-m-k_i-a\,q_i\big)\frac{\partial q_i}{\partial\omega_i} - q_i\frac{\partial m}{\partial\omega_i} + \left(m-c+\omega_i\frac{\partial m}{\partial\omega_i}\right)Q^e$$

$$= \frac{\big(P-m-k_i-a\,q_i\big)(m-c)}{a-P'(Q^e)}\left[1-\frac{1-\omega_i}{n-\Omega}\right] - q_i\frac{\partial m}{\partial\omega_i} + \left(m-c+\omega_i\frac{\partial m}{\partial\omega_i}\right)Q^e$$

$$= \frac{m-c}{n-\Omega}\left[(n-\Omega+\omega_i-1)\frac{P-m-k_i-aq_i}{a-P'(Q^e)} - q_i + (n-\Omega+\omega_i)Q^e\right]$$

$$= \frac{m-c}{n-\Omega}\left[(n-\Omega+\omega_i-1)\frac{a(Q^e-q_i)-Q^eP'+P-m-k_i}{a-P'(Q^e)} + Q^e - q_i\right]$$

Our maintained assumption that no firms exit implies that $P - m - k_i > 0$. Since $\Omega < 1$, $Q^e > q_i$, $P'(Q^e) < 0$, and $n \geq 2$, we conclude that $\partial \pi_i / \partial \omega_i > 0$. *QED.*

Proof that $\dfrac{\partial\big((1-\Omega)\,\Pi\big)}{\partial \omega_i} + \dfrac{\partial \pi_i}{\partial \omega_i} > 0$ *at* $\Omega = 0$.

$$\frac{\partial}{\partial \omega_i}\big((1-\Omega)\,\Pi\big) = \frac{\partial}{\partial \omega_i}\big((1-\Omega)\,(m-c)Q^e\big) = Q^e(T-cn)\frac{\partial}{\partial \omega_i}\left(\frac{1-\Omega}{n-\Omega}\right) = -\frac{Q^e(n-1)(m-c)}{n-\Omega} < 0$$

Using the expression for $\partial \pi_i / \partial \omega_i$ derived above,

$$\frac{\partial\big((1-\Omega)\,\Pi\big)}{\partial \omega_i} + \frac{\partial \pi_i}{\partial \omega_i} = \frac{m-c}{n-\Omega}\left[(n-\Omega+\omega_i-1)\frac{P-m-k_i-aq_i}{a-P'(Q^e)} + (1-\Omega+\omega_i)Q^e - q_i\right]$$

At $\Omega = 0$, $q_i = \dfrac{P-m-k_i}{a-P'(Q^e)}$, so

$$\frac{\partial\big((1-\Omega)\,\Pi\big)}{\partial \omega_i} + \frac{\partial \pi_i}{\partial \omega_i} = \frac{m-c}{n}\left[-\frac{(n-1)\,P'(Q^e)\,q_i}{a-P'(Q^e)} + Q^e - q_i\right] > 0. \quad \textit{QED.}$$

Characterization of when gains to trade between downstream firms exist.

Assume the total ownership by firms *i* and *j* is $\omega_i + \omega_j + \varepsilon$ and $\varepsilon > 0$. Let q_k^W denote firm *k*'s equilibrium quantity if it acquires the ε stake, and q_k^L its equilibrium quantity if it does not. Let Δ_i denote the profit difference to firm *i* between acquiring and foregoing the ε share. With these assumptions, $q_i^W - q_i^L = \varepsilon\,(m-c)\,(a-P')^{-1}$. Using the fact that $\pi_i = q_i\big(P - m - k_i - \tfrac{1}{2}a\,q_i\big) + \omega_i\,(m-c)\,Q^e$, we calculate

$$\begin{aligned}
\Delta_i &= \big(q_i^W - q_i^L\big)\Big[P - m - k_i - \tfrac{1}{2}a\big(q_i^W + q_i^L\big)\Big] + \varepsilon\,(m-c)\,Q^e \\
&= \frac{\varepsilon(m-c)}{a-P'(Q^e)}\left[P - m - k_i - \frac{a}{a-P'}\,\frac{2P-2m-2k_i+(2\omega_i+\varepsilon)(m-c)}{2}\right] + \varepsilon\,(m-c)\,Q^e \\
&= \frac{\varepsilon(m-c)}{(a-P')^2}\Big[-P'(Q^e)\,(P-m-k_i) - \tfrac{1}{2}\,a\,(2\omega_i+\varepsilon)(m-c)\Big] + \varepsilon\,(m-c)\,Q^e
\end{aligned}$$

Thus, $\Delta_i - \Delta_j = \dfrac{\varepsilon(m-c)}{(a-P')^2}\Big[(k_i - k_j)P'(Q^e) - a\,(\omega_i - \omega_j)(m-c)\Big]$ so firm *i* is willing to acquire the ε stake from firm *j* if and only if

$(k_i - k_j)P'(Q^e) - a(\omega_i - \omega_j)(m - c) > 0.$

From equations (19) and (22), $t_i(\omega_i) = c + \left(\dfrac{a[P(Q^e) - m] - k_i\, P'(Q^e) + a\omega_i\,(m - c)}{a - P'(Q^e)} \right)$, and

$t_j(\omega_j) - t_i(\omega_i) = t_j(\omega_j + \varepsilon) - t_i(\omega_i + \varepsilon)$

With this, $sign\big(t_j(\omega_j) - t_i(\omega_i)\big) = sign\big((k_i - k_j)P'(Q^e) - a(\omega_i - \omega_j)(m - c)\big)$, so firm i is willing to acquire the ε stake from firm *j* if and only if $t_i(\omega_i) < t_j(\omega_j)$. If $\omega_j > 0$, then by the definition of t_i, $t_i(\omega_i) < t_j(\omega_j)$ if and only if there exists an $0 < \varepsilon' \le \omega_j$ such that $t_i(\omega_i) < t_j(\omega_j - \varepsilon')$. This implies that firm *i* values firm *j*'s ε' stake more than firm *j* does, so a mutually profitable trade of a stake from firm *j* to firm *i* exists if and only if $\omega_j > 0$ and $t_i(\omega_i) < t_j(\omega_j)$. *QED.*

<u>*The Bertrand Fixed Proportions Mode*</u>

Proof that $\dfrac{\partial \pi_i}{\partial \omega_i} > 0$.

From the upstream first order condition (28), one can establish that

$$\frac{\partial m}{\partial \omega_i} = \frac{(m - c)\Gamma}{n - \Omega\Gamma} > 0.$$

From the definition of s_i (7),

$$\frac{\partial s_i}{\partial \omega_i} = -\frac{\partial m}{\partial \omega_i}\big(n - 1 - (\Omega - \omega_i)\Gamma\big) < 0$$

From the downstream first order condition (25),

$$\frac{\partial p_i}{\partial \omega_i} = \frac{1}{1 + (1 + a)(1 + \gamma)} \frac{\partial s_i}{\partial \omega_i} = -\frac{n - 1 - (\Omega - \omega_i)\Gamma}{1 + (1 + a)(1 + \gamma)} \frac{\partial m}{\partial \omega_i} < 0$$

From the definition of q_i,

$$\frac{\partial q_i}{\partial \omega_i} = -(1 + \gamma)\frac{\partial p_i}{\partial \omega_i} = \frac{(1 + \gamma)\big[n - 1 - (\Omega - \omega_i)\Gamma\big]}{1 + (1 + a)(1 + \gamma)} \frac{\partial m}{\partial \omega_i} > 0$$

Since $\pi_i = q_i\left(p_i - m - k_i - \tfrac{1}{2} a\, q_i\right) + \omega_i\,(m - c)Q$, and Q is invariant,

$$\frac{\partial \pi_i}{\partial \omega_i} = \frac{\partial q_i}{\partial \omega_i}\left(p_i - m - k_i - a\,q_i\right) + q_i\left(\frac{\partial p_i}{\partial \omega_i} - \frac{\partial m}{\partial \omega_i}\right) + Q\left(m - c + \omega_i\,\frac{\partial m}{\partial \omega_i}\right)$$

From the downstream first order conditions (25), $p_i - m - k_i - a\,q_i = q_i - \omega_i\,\Gamma(m-c)$

From the upstream first order condition (28), $Q = \dfrac{(m-c)\Gamma(n-\Omega\Gamma)}{1+(1+a)\Gamma}$

We can rewrite $\partial\pi_i/\partial\omega_i$ as

$$\begin{aligned}
\frac{\partial \pi_i}{\partial \omega_i} &= \frac{\partial q_i}{\partial \omega_i}\left(q_i - \omega_i\,(m-c)\Gamma\right) + q_i\left(\frac{\partial p_i}{\partial \omega_i} - \frac{\partial m}{\partial \omega_i}\right) + Q\left(m - c + \omega_i\,\frac{\partial m}{\partial \omega_i}\right)\\
&= q_i\left(\frac{\partial q_i}{\partial \omega_i} + \frac{\partial p_i}{\partial \omega_i} - \frac{\partial m}{\partial \omega_i}\right) + \frac{\partial m}{\partial \omega_i} Q\left(\frac{n-\Omega\Gamma}{\Gamma} + \omega_i\right)\\
&\qquad - \frac{\partial m}{\partial \omega_i} Q\,\omega_i\left(\frac{(1+\gamma)(1+(1+a)\Gamma)}{1+(1+a)(1+\gamma)}\,\frac{n-1-(\Omega-\omega_i)\Gamma}{n-\Omega\Gamma}\right)\\
&= q_i\,\frac{\partial m}{\partial \omega_i}\left(\frac{\gamma\left(n-1-(\Omega-\omega_i)\Gamma\right)}{1+(1+a)(1+\gamma)} - 1\right) + Q\frac{\partial m}{\partial \omega_i}\frac{\left(n-\Omega\Gamma\right)}{\Gamma}\\
&\qquad + \omega_i\,Q\frac{\partial m}{\partial \omega_i}\left(1 - \frac{(1+\gamma)(1+(1+a)\Gamma)}{1+(1+a)(1+\gamma)}\,\frac{n-1-(\Omega-\omega_i)\Gamma}{n-\Omega\Gamma}\right)
\end{aligned}$$

Since $\dfrac{\gamma\left(n-1-(\Omega-\omega_i)\Gamma\right)}{1+(1+a)(1+\gamma)} - 1 \geq -1$, $\dfrac{n-\Omega\Gamma}{\Gamma} > 1$, $q_i \leq Q$, and $\dfrac{\partial m}{\partial \omega_i} > 0$, we conclude that the sum of the first two terms is positive. $a, \gamma \geq 0$ and $n > 1$ imply that $\dfrac{(1+\gamma)(1+(1+a)\Gamma)}{1+(1+a)(1+\gamma)} \leq 1$. $\omega_i < 1$ and $1 \geq \Gamma > 0$ imply that $\dfrac{n-1-(\Omega-\omega_i)\Gamma}{n-\Omega\Gamma} < 1$. Thus, the third term is also positive and we conclude that $\partial\pi_i/\partial\omega_i > 0$. *QED.*

Proof that $\dfrac{\partial}{\partial \omega_i}\left(\pi_i + (1-\Omega)\Pi\right) > 0$ *at* $\Omega = 0$.

By the invariance of *Q*,

$$\frac{\partial}{\partial \omega_i}\left((1-\Omega)\Pi\right) = Q\left((1-\Omega)\frac{\partial m}{\partial \omega_i} - m + c\right) = Q\frac{\partial m}{\partial \omega_i}\left(1 - \frac{n}{\Gamma}\right)$$

At $\Omega = 0$,

$$\frac{\partial \pi_i}{\partial \omega_i} = q_i \frac{\partial m}{\partial \omega_i}\left(\frac{\gamma(n-1)}{1+(1+a)(1+\gamma)} - 1\right) + Q\frac{\partial m}{\partial \omega_i}\frac{n}{\Gamma}$$

So $\frac{\partial}{\partial \omega_i}\left(\pi_i + (1-\Omega)\Pi\right) = Q\frac{\partial m}{\partial \omega_i} + q_i \frac{\partial m}{\partial \omega_i}\left(\frac{\gamma(n-1)}{1+(1+a)(1+\gamma)} - 1\right)$

Since $\frac{\gamma(n-1)}{1+(1+a)(1+\gamma)} - 1 \geq -1$, $q_i < Q$, and $\frac{\partial m}{\partial \omega_i} > 0$, we conclude that $\frac{\partial}{\partial \omega_i}\left(\pi_i + (1-\Omega)\Pi\right) > 0$. *QED.*

Characterization of when gains to trade between downstream firms exist.

Using the notation introduced in the Cournot characterization,

$$\Delta_i = \pi_i - \pi_j = p_i^W q_i^W - p_i^L q_i^L - (q_i^W - q_i^L)\left[m + k_i + \tfrac{1}{2}a(q_i^W + q_i^L)\right] + \varepsilon\,(m-c)Q^e$$

From the downstream first order conditions (25),

$$p_i^W = (1+a)\,q_i^W + m + k_i - (\omega_i + \varepsilon)\Gamma\,(m-c) \quad \text{and}$$

$$p_i^L = (1+a)\,q_i^L + m + k_i - \omega_i\,\Gamma\,(m-c)$$

Substituting these into the equation above,

$$\Delta_i = (q_i^W - q_i^L)\left[(1+\tfrac{1}{2}a)(q_i^W + q_i^L) - \omega_i\,\Gamma(m-c)\right] - \varepsilon\,(m-c)\Gamma\, q_i^W + \varepsilon\,(m-c)Q^e$$

From the downstream first order conditions (25),

$$q_i = \frac{d_i + \gamma P - (1+\gamma)(k_i + s_i)}{1+(1+a)(1+\gamma)}$$

$$q_i^W - q_j^W = q_i^L - q_j^L = \frac{d_i - d_j - (1+\gamma)\left[k_i - k_j - (\omega_i - \omega_j)\Gamma\,(m-c)\right]}{1+(1+a)(1+\gamma)}$$

$$q_i^W - q_i^L = q_j^W - q_j^L = \frac{(1+\gamma)\varepsilon\,\Gamma\,(m-c)}{1+(1+a)(1+\gamma)}$$

$$\begin{aligned}
\Delta_i - \Delta_j &= (q_i^W - q_i^L)\left[(1+\tfrac{1}{2}a)(q_i^W + q_i^L - q_j^W - q_j^L) - (\omega_i - \omega_j)\Gamma(m-c)\right] - \varepsilon\,(m-c)\Gamma\,(q_i^W - q_j^W) \\
&= (q_i^W - q_i^L)\left[(2+a)(q_i^W - q_j^W) - (\omega_i - \omega_j)\Gamma(m-c)\right] - \varepsilon\,(m-c)\Gamma\,(q_i^W - q_j^W) \\
&= \frac{\varepsilon\,\Gamma\,(m-c)}{1+(1+a)(1+\gamma)}\left[\gamma\,(q_i^W - q_j^W) - (1+\gamma)(\omega_i - \omega_j)\Gamma(m-c)\right]
\end{aligned}$$

Thus firm *j* can make a mutually beneficial sale to firm *i* if and only if

$$q_i^W - q_j^W > \frac{1+\gamma}{\gamma}(\omega_i - \omega_j)\ (m-c)\Gamma \qquad\qquad QED.$$

Characterization of when firm i owning a stake instead of firm j improves total surplus.

By (26), the invariance of S implies that P is invariant. Since m can be written as a function only of Ω which is fixed, s_h and hence p_h and q_h are the same for all $h \neq i, j$ whether firm i or firm j owns the additional stake. This implies that consumer surplus and downstream firm profits in markets h are invariant to the ownership of the additional stake. Q is invariant, so $q_i + q_j$ also does not depend on the identity of the ε-acquiring firm. Since downstream demand functions are linear with slope of -1, consumer surplus in market k is $\frac{1}{2}q_k^2$. Let ΔTS denote the change in surplus due to firm i acquiring the additional stake instead of j.

$$\Delta TS = q_i^W(p_i^W - k_i - \tfrac{a}{2}q_i^W) + q_j^L(p_j^L - k_j - \tfrac{a}{2}q_j^L) -$$
$$q_i^L(p_i^L - k_i - \tfrac{a}{2}q_i^L) - q_j^W(p_j^W - k_j - \tfrac{a}{2}q_j^W) + \tfrac{1}{2}\left[(q_i^W)^2 + (q_j^L)^2 - (q_i^L)^2 - (q_j^W)^2\right]$$

By the invariance of $q_i + q_j$,

$$\Delta TS = \Delta_i - \Delta_j + \tfrac{1}{2}\left[(q_i^W)^2 + (q_j^L)^2 - (q_i^L)^2 - (q_j^W)^2\right]$$

Using $q_i^W - q_j^W = q_i^L - q_j^L$ and $q_i^W - q_i^L = q_j^W - q_j^L$,

$$\Delta TS = \Delta_i - \Delta_j + (q_i^W - q_i^L)(q_i^W - q_j^W)$$

From above,

$$\Delta TS = (q_i^W - q_i^L)\left[(3+a)(q_i^W - q_j^W) - (\omega_i - \omega_j)\Gamma(m-c)\right] - \varepsilon\ (m-c)\Gamma(q_i^W - q_j^W)$$
$$= \frac{\varepsilon\Gamma(m-c)}{1+(1+a)(1+\gamma)}\left[(1+2\gamma)(q_i^W - q_j^W) - (1+\gamma)(\omega_i - \omega_j)\Gamma(m-c)\right]$$

Thus, $\Delta TS > 0$ if and only if

$$q_i^W - q_j^W > \frac{1+\gamma}{1+2\gamma}(\omega_i - \omega_j)(m-c)\Gamma. \qquad\qquad QED.$$

References

Bergstrom, Theodore C. and Hal R. Varian (1985). "When are Nash Equilibria Independent of the Distribution of Agents' Characteristics?" *Review of Economic Studies*, 52:4 (October 1985), 715-718.

Bresnahan, Timothy and Steven C. Salop (1986). "Quantifying the Competitive Effects of Production Joint Ventures." *International Journal of Industrial Organization*, 4:2 (June 1986), 155-175.

Chipty, Tasneem and Christopher M. Snyder (1999). "The Role of Firm Size in Bilateral Bargaining: A Study of the Cable Television Industry." *Review of Economics and Statistics*, 81:2 (May 1999), 326-340.

Dasgupta, Sudipto and Zhigang Tao (2000). "Bargaining, Bonding, and Partial Ownership." *International Economic Review*, 41:3 (August 2000), 609-635.

Farrell, Joseph and Carl Shapiro (1990a). "Asset Ownership and Market Structure in Oligopoly." *RAND Journal of Economics*, 21:2 (Summer 1990), 275-292.

Farrell, Joseph and Carl Shapiro (1990b). "Horizontal Mergers: An Equilibrium Analysis." *American Economic Review*, 80:1 (March 1990), 107-126.

Flath, David (1989). "Vertical Integration by Means of Shareholding Interlocks." *International Journal of Industrial Organization*, 7:3 (September 1989), 369-380.

Gilo, David (2000). "The Anticompetitive Effect of Passive Investment." *Michigan Law Review*, 99:1 (October 2000), 1-47.

Greenhut, M.L. and H. Ohta (1979). "Vertical Integration of Successive Oligopolists." *American Economic Review*, 69:1 (March 1979), 137-141.

Joskow, Paul L. and Jean Tirole (2000). "Transmission Rights and Market Power on Electric Power Networks." *RAND Journal of Economics*, 31:3 (Autumn 2000), 450-487.

O'Brien, Daniel P., and Steven C. Salop (2000). "Competitive Effects of Partial Ownership: Financial Interest and Corporate Control." *Antitrust Law Journal*, 67:3, 559-614.

Pitofsky, Robert (1997). "Vertical Restraints and Vertical Aspects of Mergers – A U.S. Perspective." http://www.ftc.gov/speeches/pitofsky/fordham7.htm.

Raskovich, Alexander (2000). "Pivotal Buyers and Bargaining Power." *Economic Analysis Group Discussion Paper 00-9*, US Department of Justice.

Reitman, David (1994). “Partial Ownership Arrangements and the Potential for Collusion.” *Journal of Industrial Economics*, 42:3 (September 1994) 313-322.

Reynolds, Robert J. and Bruce R. Snapp (1986). “The Competitive Effects of Partial Equity Interests and Joint Ventures.” *International Journal of Industrial Organization*, 4: (June 1986), 141-153.

Riordan, Michael H. (1991). “Ownership without Control: Toward a Theory of Backward Integration.” *Journal of the Japanese and International Economies*, 5:2 (June 1991), 101-119.

Riordan, Michael H. (1998). “Anticompetitive Vertical Integration by a Dominant Firm,” *American Economic Review*, 88:5 (December 1998), 1232-1248.

Salinger, Michael A. (1988). “Vertical Mergers and Market Foreclosure.” *Quarterly Journal of Economics*, 103:2 (May 1988), 345-56.

Salinger, Michael A. (1989). “The Meaning of ‘Upstream’ and ‘Downstream’ and the Implications for Modeling Vertical Mergers.” *Journal of Industrial Economics*, 37:4 (June 1989), 373-387.

Seade, Jesus K. (1980). “The Stability of Cournot Revisited.” *Journal of Economic Theory*, 23:1 (August 1980), 15-27.

Spengler, Joseph J. (1950). “Vertical Integration and Antitrust Policy.” *Journal of Political Economy*, 58:4 (August 1950), 347-352.